Math 132
Homework 4 Comments

I graded 4 of the problems:
Section 3.1: 3
Section 3.2: 1d
Section 4.1: 2c, 5

Each problem is worth 3 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 3 indicates a correct or nearly correct solution. Otherwise the grade given is 1 or 2 depending upon how much work was put in and how close the solution is to being correct. How neat and clear your solution is also affects the grade I give.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and return the homework in the next section.

The following are comments and occasionally solutions for the graded problems.

General Comments
The maximum number of points was 12. The high score was 12, and the median was 9. There were no graphing problems this time and it was a fairly short and easy assignment, so not surprisingly the scores were higher than on previous problem sets.

3.1 3. I did this problem in section but it still seemed like the best to grade out of this section of the book. Parameterize the quarter circle in three parts, one along the x-axis from $(0,0)$ to $(1,0)$, one along the quarter circle from $(1,0)$ to $(0,1)$, and one along the y-axis from $(0,1)$ to $(0,0)$. Then the integral of x^2 along the first segment will be 0 since dy is 0, and the integral along the third segment will be 0 since x is 0 at every point. Some people didn't mention this and I didn't take off for it, but be careful on the final.

For the second segment, parameterize it by $x = \cos\theta, y = \sin\theta$ where $0 \le \theta \le \pi/2$. We have $\int_0^{2\pi} cos^3\theta \, d\theta = \int_0^{2\pi}(1 - sin^2\theta)\cos\theta \, d\theta$; you can do substitution now and work out the details. The answer is 2/3.

We now use Green's Theorem, noting that $P(x,y) = 0$ and $Q(x,y) = x^2$. So we have $\iint_D 2x \, dx \, dy$. Parameterize the region now by $x = r\cos\theta$ and $y = r\sin\theta$ where $0 \le r \le 1$ and $0 \le \theta \le \pi/2$. This gives $\int_0^{2\pi}\int_0^1 2r^2\cos\theta \, dr \, d\theta = 2/3$ as you can easily work out.

3.2 1. I graded part (d). The first step is to determine whether or not $y\,dx - x\,dy$ is independent of path. Some people just claimed it wasn't, and although integrating it around a closed path and not getting zero justifies that, I wanted to see some understanding that a differential is independent of path iff it is exact iff it is closed, for a star-shaped domain which certainly $\mathbb{R}^2$ is. Therefore for such a domain it is sufficient to check whether or not the differential is closed. We compute $\frac{\partial P}{\partial y} = 1$ and $\frac{\partial Q}{\partial x} = -1$, and these two are not equal so the differential is not closed and thus not independent of path.

We now have to find a closed path γ around which the integral is not zero. Lots of paths work and I was happy to see people try out several different possibilities. The unit square whose lower left corner is at the origin is one nice path to use; you have to do four cases but two of them are 0 and the other two add up to -2 as you can check. Perhaps the easiest path to use was the unit circle. We parameterize as usual by $x = \cos\theta, y = \sin\theta$ and get $\int_\gamma y\,dx - x\,dy = \int_0^{2\pi}(-\sin^2\theta - \cos^2\theta)\,d\theta = -2\pi$.

4.1 2. I graded part (c). Many people did not seem to notice that $|dz|$ is defined in the text on page 104 as $|dz| = \sqrt{(dx)^2 + (dy)^2}$. Gamelin then goes on to derive $|dz|$ in polar coordinates, which some people copied without trying to understand what was going on. Let's try to do it carefully. We are integrating around the unit circle, so it's natural to parameterize the points according the angle θ; the radius is of course 1. So we have $x = \cos\theta$ and $y = \sin\theta$. This gives $dx = -\sin\theta\,d\theta$ and $dy = \cos\theta\,d\theta$, and it follows $|dz| = d\theta$.

We have $\int_\gamma z^m \, |dz| = \int_0^{2\pi}(\cos\theta + i\sin\theta)^m \, d\theta = \int_0^{2\pi} e^{im\theta}\,d\theta$. Now we can either peek ahead to the next section and note that we can integrate this directly as a complex function (for $m \neq 0$) as $\frac{1}{im}\, e^{i\theta m}\Big|_0^{2\pi} = 0$, or we can be integrate this as a pair of real functions by noting that

$\int_0^{2\pi} e^{im\theta}\, d\theta = \int_0^{2\pi} (\cos m\theta + i \sin m\theta)\, d\theta = \frac{1}{m}(\sin m\theta - i \cos m\theta)\Big|_0^{2\pi} = 0$. I gave full credit for either way, although you had to show some work and not just state the answer (which was in the back of the book). You also needed to handle the case $m = 0$ separately, which is easy: $\int_0^{2\pi} e^{i0\theta}\, d\theta = \int_0^{2\pi}\, d\theta = 2\pi$.

4.1 5. I did this in section but it was the only ML-estimate problem on the homework so I wanted to grade it so that everyone can make sure they understand it. To apply the ML-estimate, we need to determine the length L of the curve γ we're integrating over, and then bound the function we're integrating, showing it has value M or less on all points of γ. First note that γ is the circle of radius 1 centered about the complex point 1, and thus $L = 2\pi$. We just need to show that $\left|\frac{e^z}{z+1}\right| \leq e^2$ for all points on this circle (note that older printings of the book have $z + 3$ in the denominator; this is fine too). If z is on γ, then $|z + 1| \geq 1$ as you can see geometrically, or more carefully by noting $|z + 1| = \sqrt{(x + 1)^2 + y^2} \geq 1$ since both $x \geq 0$ and $y \geq 0$ on γ, where $z = x + iy$. It follows $\left|\frac{e^z}{z+1}\right| \leq |e^z| = |e^x|\left|e^{iy}\right| = e^x$. But for z on γ we clearly have $0 \leq x \leq 2$, so $e^x \leq e^2$ which is what we wanted to show.